Geomorphologic Modelling of the Interaction between River Flow, Sediment and Vegetation

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ABSTRACT

Although sediment and vegetation processes interact, they respond individually to flow characteristics at different spatial and temporal scales. A hierarchical modelling strategy is proposed. Separate models describe vegetation and sediment dynamics at their appropriate scales, with their interaction described through feedback between the models. A Cellular Automaton (CA) describes how sediment is transported through a river, represented by a lattice of cells, and predicts the volume of sediment stored in a cell. The sediment distribution obtained from the CA model describes the habitat for vegetation. A Fuzzy Rule-Based model simulates vegetation responses to flow at a yearly interval and the outputs are applied to the habitat. The new vegetation state then influences sediment behaviour in the next episodic CA model application. The scales of the models are linked temporally through flow characteristics and spatially by geomorphological organisational units.

Keywords: *Geomorphology; Hierarchical Model; Cellular Automata; Fuzzy Logic.*

1 INTRODUCTION

In a river, the local hydraulics, channel form, and instream vegetation are interdependent. Vegetation and channel form determine hydraulic conditions for a given discharge; hydraulic conditions and channel form define habitat for vegetation establishment and growth; vegetation and hydraulic determines channel form by controlling the movement, trapping, and storing of sediment. Effective environmental management requires the ability to predict responses of river processes and in particular the responses due to the mutual feedback between vegetation and sediment organisation (Jordanova and James, 2003). Sediment organisation however, responds to discrete flow events over small distances whereas changes in vegetation patterns, over large areas, occur as a result of seasonal variations in the flow regime. A predictive model must therefore account for processes over a range of time scales.

This study employs a modelling approach that is based on the tendency of natural systems to self-organise in scale hierarchies. Separate models for describing vegetation and sediment dynamics at their appropriate scales are integrated such that their interactions are described through feedback between the models. This geomorphological model intends to allow river managers in Kruger National Park, South Africa to make decisions based on the model's predictions. Since reeds play such a large primary role as a modifier of geomorphology of the rivers in the Kruger National Park, a model for describing the change in reeds on sand bars based on the flow regime has been developed. The reed model is a Fuzzy Rule-Based model and makes predictions based on rules relating reeds with flow characteristics.

A Cellular Automaton (CA) model is used to describe how sediment is organised when a river's velocity regime is applied to a series of cells representing the river bed. The CA model is based on the ability of rivers to store sediment. Sediment is stored in association with fixed form of the channel for a given velocity regime. When a different velocity regime is applied to the cells a change in storage is caused and sediment is transported downstream until it reaches an area where the full capacity for storage (ultimate storage state) has not been reached. Such an area would then be able to accumulate sediment until the ultimate storage state is reached. However, long-term changes to the ultimate storage state are occurring as a result of changes in channel form. Thus, changes in the flow regime and channel form would produce a continual change in the velocity regime. This induces ongoing positive and negative feedback loops on the ultimate storage state, which will produce a dynamic riverbed. There is also feedback from vegetation on the ultimate storage state through slowing down velocities of water flowing through the vegetation. In this way vegetation produces a more gradual influence on the state of sediment organisation within a river through reed growth and reed cover expansion. This influence of vegetation on river geomorphology is too significant to ignore when simulating channel changes over a time scale of years to decades. Thus, to predict river geomorphology more accurately, the riparian vegetation patterns also have to be predicted. In turn, the patterns of reed growth are determined by availability of substrate or sediment deposits. This cause and effect relationship is catered for, through a hierarchical modelling structure by linking the models temporally through flow characteristics and spatially by geomorphological organisational units.

2 HIERARCHICAL MODELLING

A system can be modelled across a broad range of scales using a hierarchy of dynamically uncoupled models. This hierarchical modelling methodology is constructed so that it is objective and testable and is based on properties of non-linear, open systems, with no new principles introduced. By isolating dynamics at different scales, better predictability is achieved with hierarchical modelling than in models, in which scales are dynamically mixed (Werner, 1999).

A hierarchy is a formal organisation of various spatial or temporal sizes or levels graded from small to large. Hierarchies are useful devices for organising information and analysis across multiple scales. According to hierarchy theory, nature can be partitioned into naturally occurring levels that share similar time and space scales, and that interact with higher and lower levels in systematic ways (Jewitt and Gorgens, 2000). One end of the spectrum is characterised by high perspective and low detail and the other end, by high detail and low perspective.

Movement towards a lower hierarchical level results in process details becoming clearer, while assemblages, patterns, and relationships formed by these become less noticeable. Movement in the opposite direction, towards a higher perspective region, results in the opposite effect; assemblages, patterns and relationships emerge while the detailed processes that form them become less clear (Jewitt and Gorgens, 2000). Viewing the system from several vantage points over the entire spectrum provides complementary understandings of real world systems whose complete nature is beyond human perception. This integrated approach is consistent with a natural way of describing different processes (Aassine and El Jai, 2002).

In rivers there is a mutual interaction between geomorphological processes and vegetation processes that requires feedback coupling in a hierarchical modelling framework. Feedback is associated with the interaction between small-scale variables and large-scale variables since processes at different scales affect each other. Because of the existence of different dominant processes at various scales, the effects of interacting processes need to be incorporated to obtain patterns that span over a wide range of scales. Patterns forming at slow and large scales are constructed and organised by the interaction of many small and fast processes (Zhang et al, 2004).

Vegetation has to adjust to the river environment but vegetation itself affects river hydrodynamics and morphodynamics through effects on hydraulic roughness, sediment erodibility and sediment trapping. It is recognised that the increase of riverine vegetation will create further opportunities for sediment to be trapped, which in turn provides habitat for riverine vegetation growth. The geomorphology of natural river channels is also determined by the interaction of the river flow with the sediment in the channel. This nonstatic property of river channels means that there is a complex interdependence between flow characteristics and the form of the channel (Michaelides and Wainwright, 2004). Hence, there are several feedback cycles that affect the overall natural development of river geomorphology. When looking at these cases, it is important to examine the temporal and spatial scales of the mutually interacting processes.

Different processes are primarily dominant at different scales. This means that correlations derived at one scale might not be applicable at another (Zhang et al, 2004). In practice the effects of geomorphological processes occur on scales much smaller than those relevant to vegetation interactions. For example, vegetation models related to geomorphology would require explicit models investigating the development of bars and channel changes on characteristic scales in the order of 100 metres and months or years. Where models of fluvial sediment organisation use relationships for sediment transport on the scales of metres and seconds.

Vegetation models are parameterised in order to compensate for detailed analyses or models of the smaller-scale processes. However, a model at a smaller-scale can be incorporated to account for the reactions of vegetation that arises from the collective behaviour of the smaller-scale sediment organisation processes. These reactions could not be readily predicted directly from examining the parameters that govern the interactions of vegetation.

Werner (1999) is in favour of this explicit hierarchy of models operating at different scales, with minimal information exchanged between them, with only a few crucial aspects of the dynamics at the next level down are incorporated in each model. In this approach the most direct influences on larger-scale behaviour are caused by interactions between variables, which have scales that are not greatly smaller than those of the phenomena of interest (Murray, 2003).

River systems are inherently complex due to the large number of interconnected parts. The interactions and feedback occurs over a range of time and space scales (Michaelides and Wainwright, 2004). The notion of complexity is closely tied up to with the concept of scale. From this perspective, the assumption is that even the most complex of systems, when viewed at a component scale somehow becomes simpler, and thus more mathematically tractable. To ignore the complex within-system interactions, which give rise to that self-organisation, is to neglect an integral underpinning (Favis-Mortlock, 2004).

A hierarchical modelling structure would be simpler. As systems become more complex, constructing models requiring parameters to compensate for the smaller-scale processes, or controlled measurements, can become much more difficult. The proposed method allows models to be integrated without sacrificing the required detail that is needed to make accurate predictions. The advantage of these simple models is that it is often straightforward to understand how they work (Werner, 1999).

Vegetation is not easily modelled by working from equations representing the vegetation dynamics at a broad scale, and even at the scale of individuals and species, equations governing the interactions have not generally been established. These systems are often modelled using a rule-based framework (Murray, 2003). Rule based models are consistent with simplification of the input requirements at broad spatial and temporal scales as used in coarse resolution models or simple pragmatic models. The effects of local heterogeneity are averaged out at broad scales, and consequently, patterns appear to be more predictable at broader scales (Wiens, 1989).

Combining rule based models with a sediment organisation models creates difficulties in that these rules are determined by experts is at a scale which is not compatible with the commonly used equations used in sediment organisation models. As a result these rules employ variables describing sediment organisation that are not based directly on the underlying processes or measurements. Models at broader scale patterns deliver less predictive accuracy at specific points in pace and time (Costanza and Maxwell, 1993).

Vegetation responses are observed at the spatial and temporal scales considered important by the experts studying vegetation growth phenomena. Changing the scales of processes observed by experts results in an increase in uncertainty in the rules since the rules that are described by these processes are observed at these time scales. If the sediment organisation model were at a higher scale the output would be more descriptive but it would also increase the difficulty to model accurately the available substrate as required for the vegetation model. It is also not practical to model vegetation at time steps as small as hours because the changes would be too small. At these time scales, sensitivity analyses shows that many processes can be neglected, since vegetation growth would appear stagnant (Jewitt and Gorgens, 2000). This creates the problem in which the most appropriate modelling strategy in terms of the range of processes and scales involved is not clear.

The scales of the processes forming the environment for vegetation and the vegetation processes are rarely the same. The need to incorporate multiple process models in a hierarchical modelling system implies a need to incorporate multiple scales of resolution, both spatial and temporal. The hierarchical modelling structure described, therefore, needs to operate at spatial scales that allow simulation of the processes affecting responses at the scale at which those processes occur, as well as being able to output information at the observation scale of the vegetation response patterns. Thus, linking of abiotic components to a biotic response requires the ability to simulate at varying spatial and temporal scales.

3 Cellular automaton modelling

Self-organisation of sediment organisation is modelled with the use of cellular automaton (CA). The developed CA model represents a space over which water flow occurs. This space is divided up into a series of cells. These are usually part of a rectangular grid. The rules and relationships that are used in the model are then applied at the scale of individual cells. Interactions are between adjacent cells.
The rules and relations may be viewed as positive and negative feedbacks and thus each cell may be seen as an automaton with its behaviour controlled by the positive and negative feedbacks to which it is subject (Favis-Mortlock, 2004). The CA model then self organizes and gives rise to larger patterns on the cellular grid.

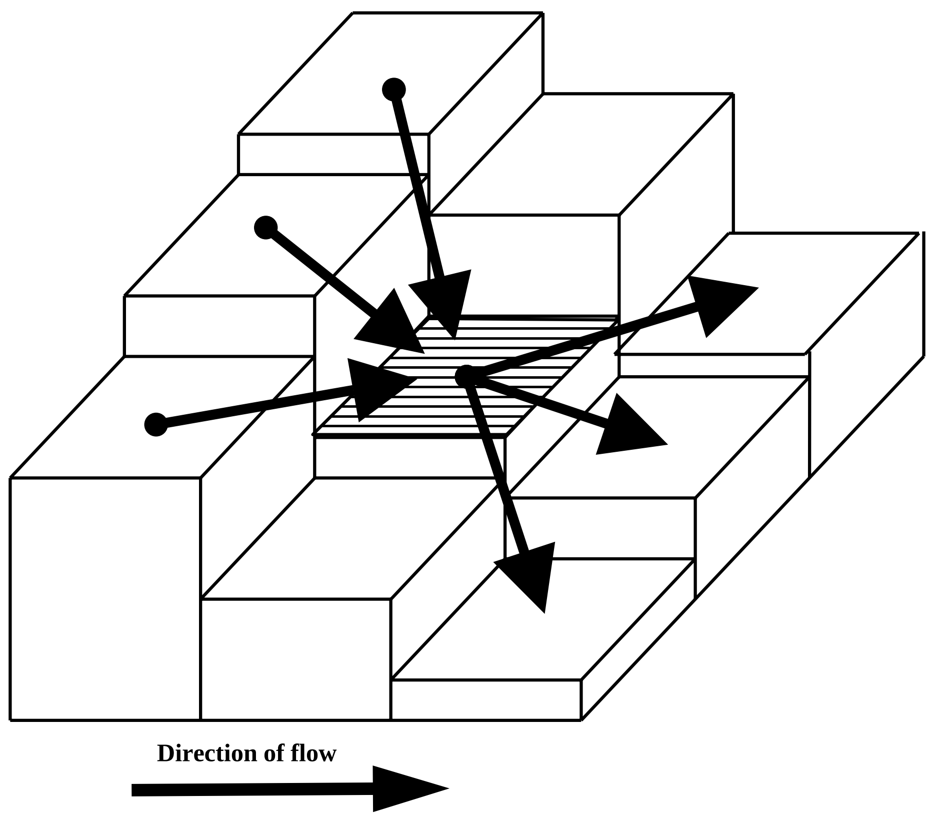

Figure 1: Movement of sediment through a cell in a grid

3.1 Sediment routing

A certain volume of sediment is fed into the first row of cells representing the upstream end of the river. The amount of sediment fed into the model depends on the sediment inflow rate and the time step that is specified by the user. This sediment moves cell-by-cell according the relationships for sediment flow between cells. The outflows from upstream cells are the inflows to the downstream cells (Figure 1).
An iteration begins when sediment enters the upstream end of the cellular grid and ends when outflow has been calculated for the cells at the downstream end of the grid. There is no net loss or gain of sediment, as sediment remains budgeted for during simulation.

3.2 Sediment allocation according to local slope

The amount of sediment that will flow from the upstream cell to a downstream depends on the fraction between the local slope between the upstream and downstream cell and the sum of the slopes of all the downstream cells to which sediment would flow. When considering the three downstream cells to which sediment may possibly flow, a cell with lower downstream cell would a have a larger slope and as a result will also have a larger fraction of the total outflow from the upstream cell allocated to it. Cells that are not immediate downstream neighbours are at an angle of 45 degrees to the overall downstream direction and would have a larger horizontal distance from the upstream cell. In real rivers, water can flow uphill when the surface slope is positive if there is enough momentum (Murray and Paola, 1994). This means that if one or more of the slopes are negative, flow of sediment may still occur. The model takes account of this by allowing some fraction of sediment outflow from the upstream cell to possibly flow to the downstream cell with a higher elevation.

3.3 Sediment storage

It is assumed that the volume of sediment that can potentially be stored in a cell is related to velocity in that cell. It is therefore possible to assign an ultimate stable storage state ($\mathbf{S}_{uss}$) for all possible velocities (James et al, 2001). This relationship is showed in Figure 2.

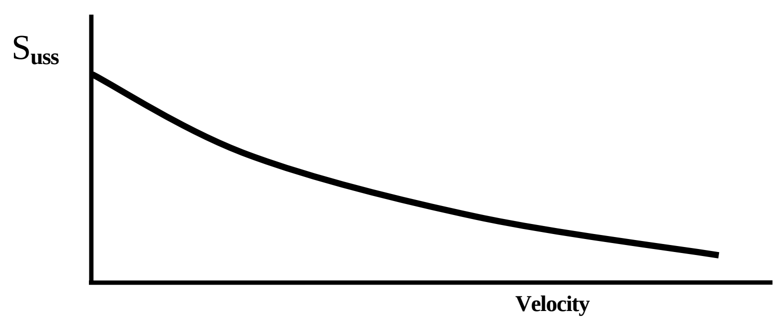

Figure 2: Ultimate sediment storage as a function of velocity

The volume of sediment that will be deposited depends on S_{uss} as a function of velocity for each cell. If the velocity in the cell is slower, the amount of potential sediment storage will be larger. S_{uss} of each cell is measured from the surface of each the three downstream cells. Depending on the relationship between the sediment inflow, the S_{uss} and the difference in elevation between upstream and downstream cells, a value for the full possible outflow to a downstream cell is determined. If the elevations of the three different downstream cells were all dissimilar, the values for each of the full possible outflows into each of the downstream cells would also be dissimilar.

Rules for sediment outflow are specified for six possible cases. The cases are shown in Figure 3. Dissimilar elevations of upstream and downstream cells suggest dissimilar cases applying. Each outflow case would specify a different full possible outflow to the downstream cells, which is proportioned according to local slope to obtain the actual outflow from the upstream cell to the downstream cell.

After an iteration the angle or repose rule is implemented (Nield et al, 2002). This allows avalanches to occur such that height differences between cells are not higher than what the angle of repose stipulates. Each cell collapses allowing sediment to move to neighbouring cells according to the slopes between neighbouring cells. This continues until no further change in the sediment pattern can be seen. At this point, the system is said to have reached a self-organised pattern.

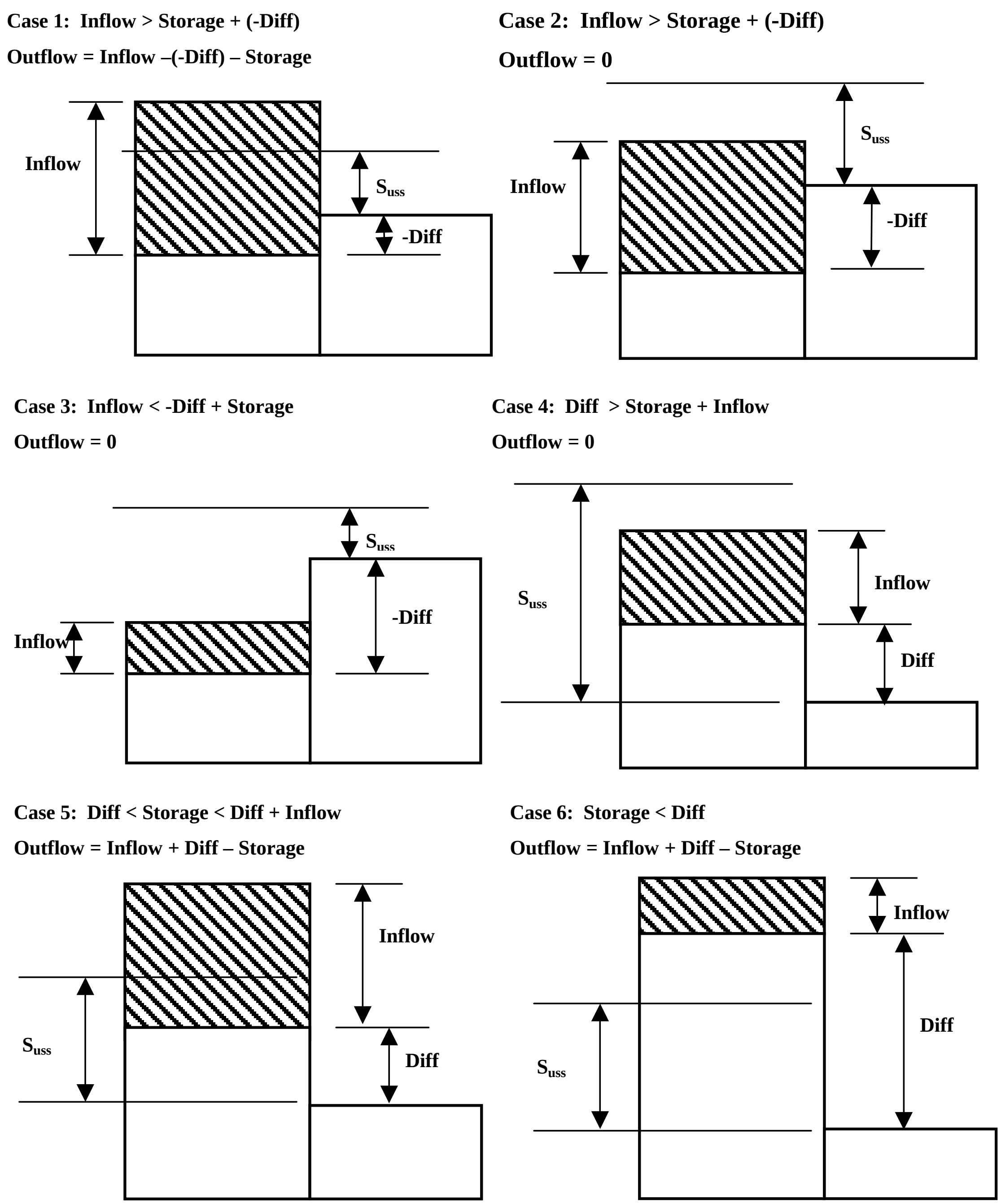

Figure 3: Outflow case for when the difference between upstream and downstream cells are less then zero

4 FUZZY RULE-BASED MODEL

Reed rhizomes are transported to alluvial bars and then establish themselves by clonal reproduction and set up a network of rhizomes, which cover the bar. Reed cover expansion is the processes of bar colonisation. The proposed Fuzzy Rule-Based model of reed dynamics simulates the expansion of reed cover on sand bars in a river. The model runs at an annual time step and determines the magnitude of the percentage of change of the area that could possibly be covered by reeds. The model assumes hypothetical relationships between reeds and flow regime (Figure 4). The input variables for the reed are Floods, Flow variability, Proportion of medium flow, Reed Age and winter flow conditions.

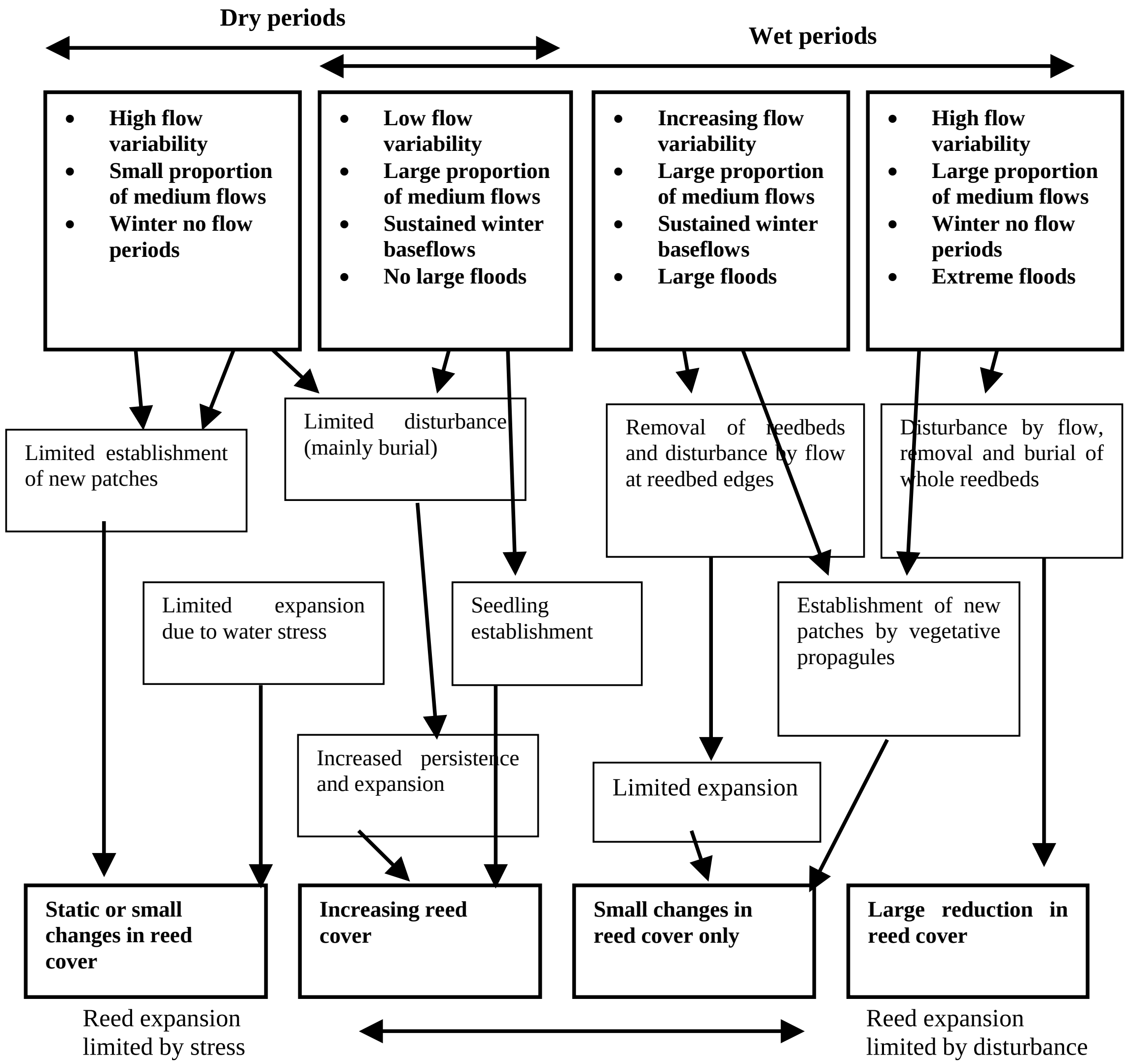

Figure 4: Conceptual diagram showing hypothetical relationships between flow conditions and changes in reed cover (Kotschky, 2003)

4.1 Floods

In the model, it is required to specify the biggest flood event for the year. Floods cause disturbances to reeds through the removal or burial of reed patches. Reeds are however, fairly resistant to flooding and will not be destroyed. Floods may break reeds and carry them away. Extreme floods destroy established riparian vegetation.

4.2 Proportion of medium flows

Monthly flows are input into the model and to determine the proportion of medium flows. Medium flows are a range of flows specified by the user that falls near to the average monthly flow for the river. Long lasting flows can cause anaerobic conditions to develop in the roots, and this affects the process of nutrient uptake.

4.3 Flow variability

The coefficient of variation (COV) is computed for the monthly flow inputs. The resultant COV is then compared to the COV for monthly flows obtained from earlier flow records to determine to what extend the flow is variable. Reeds require periodic inundation to supply water and nutrients to alluvial substrate of the bar. Low flow variability implies that flows will invariably be low and will affect the reeds though lack of water on higher elevations. This will limit expansion of reed cover because the health of the reeds will be undermined. High flow variability plays a role in reed expansion by supplying reeds on higher elevations with water. It is also very important for seedling establishment on higher elevations.

4.4 Sustained winter base flow.

It is also necessary to state whether winter base flows will be sustained. This is required for determining the amount of water stress that the reeds will endure during dry periods.

4.5 Reed age

Phragmites rhizomes are transported to alluvial bars and then establish themselves by clonal reproduction and set up a network of rhizomes, which cover the bar. These then grow normally during the dry season to young stems. Young stems do have aerial shoots that are more flexible then mature reed stems and the density of stems are sparse. Once maturity has been reached, the stems are stronger and the number of stems per unit area is higher. Mature reeds are resistant to flooding and will not easily be destroyed (Nicolson, 1999). The states are as follows:

- Rhizomes – No aerial shoots growing
- Young stems – Aerial shoots that are more flexible and the density of the reed bed is lower.
- Mature – Fully developed high density reed bed causing leaves growing on the stems.

4.6 Rule-base

The relationships shiown in Fig 4 can be expressed as rules describing reed growth dynamics. Linguistic variables provide a language for the expert to define rules to form the knowledge base of the system. The information that is used to describe change in reed cover has linguistic variables, such as “negative small change” or “positive large change” of reed cover. The rules are usually of form similar to the following: “if **A**, then **B**” where **A** and **B** are fuzzy membership functions which in turn specifies to what degree a statement would be true (Klir, 1995). **A** forms the condition that describes to what degree the rule applies, while the conclusion assigns a membership function to the output variable.

Depending on the number and characterisation of different physical attributes, sets of rules can be defined that describe the entire available physical template as combinations of physical attributes and then define conclusions for each of the possible combinations. The number of rules resulting from this depends also on the number of linguistic variables used for each of the input variables. The rules allow precise as well as imprecise information as input data to be processed (Klir et al, 1987).

4.7 Fuzzy logic

Fuzzy logic has been implemented into the rule-based model in order to deal with the inherent uncertainty that exists when developing rules for the model. Fuzzy logic deals with the concept of partial truth or truth-values between "completely true" and "completely false" (Bezdek, 1993). Dealing with uncertainty is crucial for modelling the complexity of real systems such as rivers. Fuzzy models allow working with imprecise or “fuzzy” information and exist in the form of fuzzy expert systems (Kaufmannand and Gupta, 1985). They can take advantage of expert knowledge that is readily available from experienced scientists.

5 HIERARCHICAL MODEL LINKAGE

For the purposes of the sediment model the spatial scale is of the order of metres or smaller and time steps for the sediment model are of the order of hours. The reed model, on the other hand, has a temporal time scale of one year and will spatially be at the so-called “geomorphological unit” scale. Geomorphological units are river features which are characterised by a spatial scale of metres to tens of metres (e.g. bars pools and bedrock outcrops). The connection between sediment organisation and riverine vegetation is made at the geomorphological unit scale. In this case, the vegetation establishes and grows on sand bars, which are produced by the sediment model due to sediment organisation patterns that results from river flow. Effects of the fundamental phenomena at the sediment organisation scale therefore need to be upwardly integrated to provide useful information and relationships at the geomorphological unit scale appropriate for vegetation modelling. The substrate available for expansion of reed cover would be presented in the sediment organisation model as sediment bars, which are not inundated during the dry season. The change in the reed cover as predicted by the reed model would then be implemented to each of these exposed bars. Since the reed models runs at the geomorphic unit scale, each bar would be updated individually according to the initial reed age and extent of reed cover.

Temporally the two models will be linked through hydraulics. It is necessary to note that what appears to be noise at a higher level could be tuned into significant perturbations on the lower level. The sediment organisation model requires flow hydrographs to simulate sediment interactions. Consequently the hydraulic input into the reed model will be monthly average flows for a given year. The reed model does not require detailed hydraulic information in order to make accurate predictions.

The trends of flow hydrographs would only appear as noise at the time scales of the reed model. It is also useful to consider the dynamics of sediment at an event scale. Sediment transport is episodic in nature and most of the transport occurs during high flow events. This means that the vegetation state can be considered unchanged throughout a flow event. Flood events cause large amount of vegetation to be washed away.

5.1 Linkage procedure

Sandbars at the geomorphological unit scale are simulated through sediment organisation resulting in cells in the CA model to have sediment elevations that are higher than the water surface at low flow conditions. The linkage procedure occurs after every time step in the reed model.

1) The numbers of cells with reeds in the different age categories occupying cells are counted. The reason for this is that each reed age category induces a different percentage change in reed cover. The fraction of cells for each age category to the total number of vegetated cells are then multiplied with each of the percentage change in reed cover obtained for the respective age categories and then summed to obtain the total increase or decrease in the number of vegetative cells.
2) If there is a decrease in reed cover, the number of cells vegetated by reeds is multiplied by the percentage change in reed cover.
3) The reed age is updated: The Rhizomes become Young stems, and Young stems become Mature reeds.
4) The cells, which have an elevation higher than winter base flow and does not already have vegetation established on it, are grouped and counted. This is required in order to determine how many cells are suitable for reed growth on a specific grouping of elevated cells. The percentage of increase in reed cover is then multiplied with these cells.
5) The percentage change in reed cover as fractioned according to the different reed age categories is implemented to decrease or increase the vegetated cells. If there is an increase the new vegetated cells will be rhizomes.

5.2 Modelling results

Figures 5-8 shows the way the two models interact. Cells are colour coded in order to indicate special features.

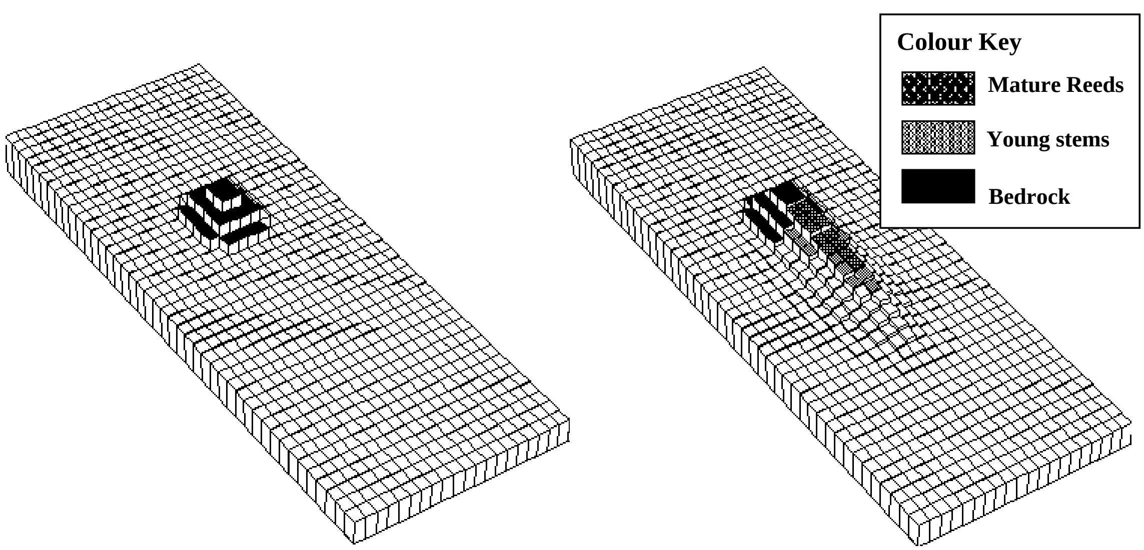

Figure 5: Bedrock causes sediment to accumulate on which vegetation growths.

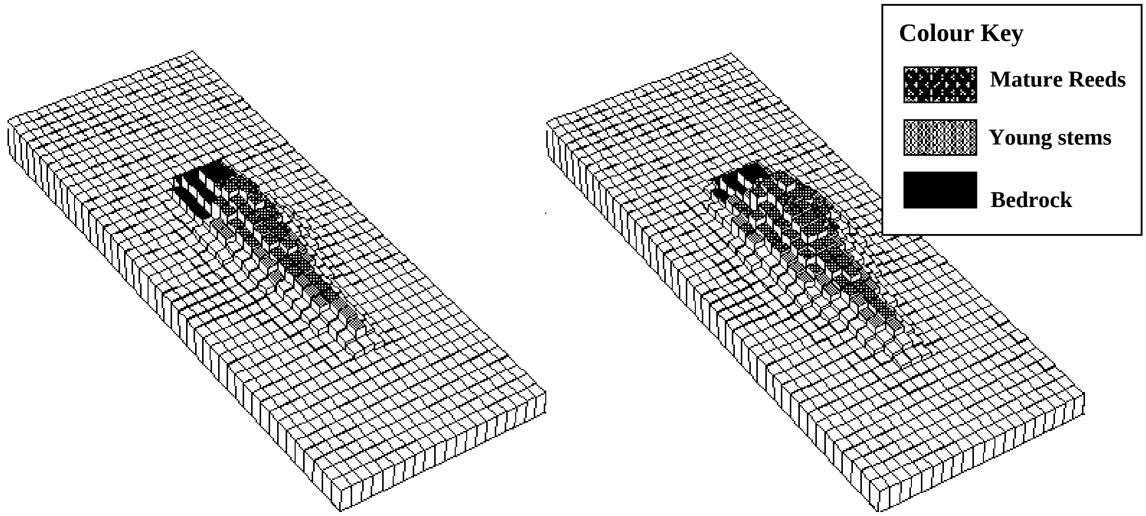

Figure 6: The riverine vegetation causes velocities to be slowed down and cause more sediment to be stored within the cells.

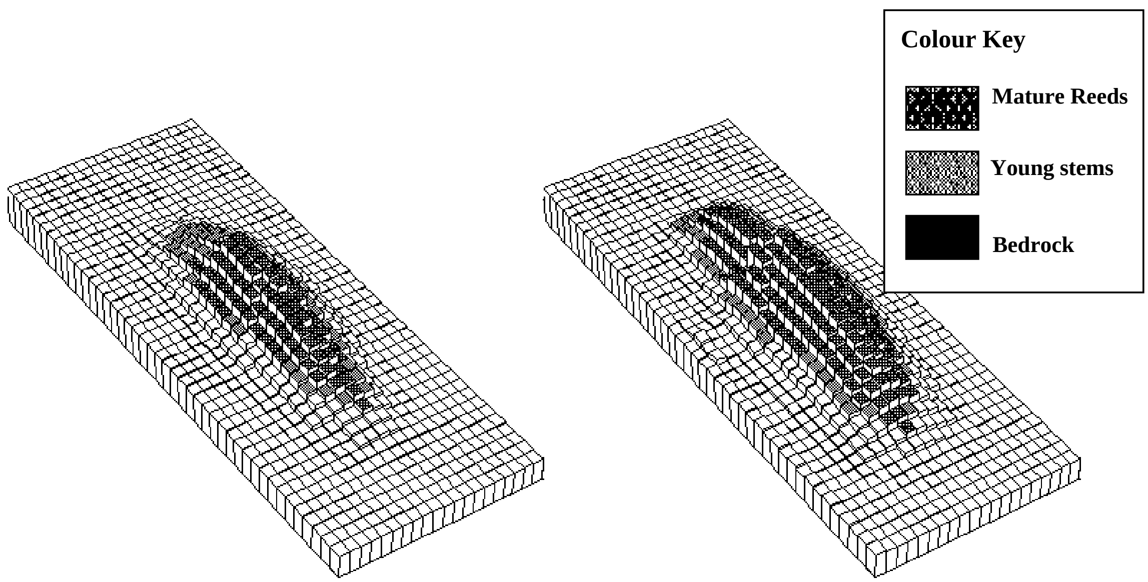

Figure 7: An increase of vegetation creates further opportunities for sediment to be stored, which in turn provide further habitat for vegetation to grow.

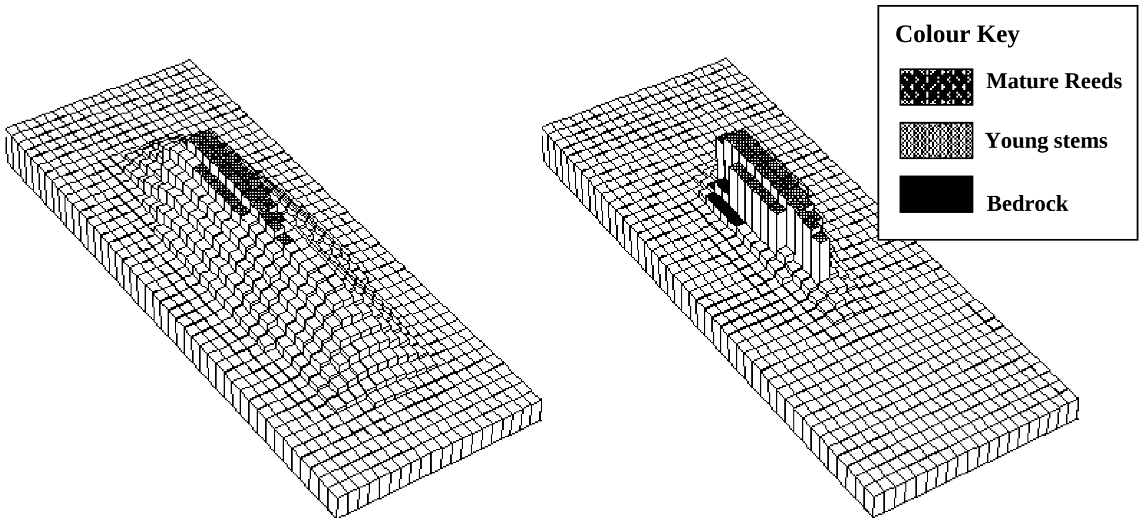

Figure 8: Large flood events cause large amount of vegetation to be washed away. In the CA model, reeds would be considered as already been removed so that sediment processes can be modelled without regard for reeds that would be washed away.

6 CONCLUSION

Hierarchical modelling can be achieved through integrating models at their appropriate spatial and temporal scales. A Cellular Automaton (CA) model has been linked with a Fuzzy Rule-Based model in two ways. First, through temporally relating the time unit, for the hydraulic regime, appropriate for the individual modelling scales; and secondly, by relating a smaller spatial unit to a larger one. The hydraulic regime for the reed model is specified in terms of monthly averages and flood flow magnitudes whereas the sediment model requires velocities profiles due to the flow hydrograph from the same year. The cells within the CA forms organised patterns when a velocity field is applied to the cells. These organised patterns are related to geomorphological units resembling sandbars on which reeds grow.

The reed model specifies a change in the reed cover after a yearly time step. The change in vegetation cover would change the velocity profile for a given discharge. As discharge changes incrementally with each time step in the CA model, velocities increase or decrease causing a change in the volume of sediment that cells have in storage. Cells with higher elevations form organised patterns due the velocity profile. These organised patterns are recognized as sandbars acting as substrate for vegetation. A reed model has been developed to determine a single numerical value for the change in reed cover, which is applied to a singular sandbar produced by the CA model.

REFERENCES

Aassine, S. and El Jai, M. C. 2002. Vegetation dynamics modelling: a method for coupling local and space dynamics. Ecological modelling 154, 237 – 249.

Bezdek, J C 1993. Fuzzy Models What Are They, and Why?, IEEE Transactions on Fuzzy Systems, 1:1, 1-6.

Costanza, J. R. and Maxwell, R. 1993. Resolution and predictability: An approach to the scaling problem. Landscape ecology. Vol. 9, 47 – 57.

Favis-Mortlock, D. 2004. Self-Organisation and cellular automata models In Environmental modelling: Finding simplicity in complexity. Edited by J Wainwright and M Mulligan John Wiley & Sons, Ltd.

James, C. S., Odiyo, J. O. and Lambert, M. F. 2001. Modelling sediment movement and storage in semi- arid, bedrock controlled rivers. 7th International Conference on Fluvial Sedimentology, Lincoln, Nebraska, U.S.A., August 5-10, 2001.

Jewit, G.P.W. and Gorgents, A. H. M. 2000. Scale and model interfaces in the context of integrated water resources management for the rivers of KNP. Water Research Commission Report No 627/1/00.

Jordanova, A. A. and James, C. C. 2003. Experimental Study of Bed Load Transport through Emergent Vegetation get reference. Journal of Hydraulic Engineering Vol. 129 No. 6, 474 – 478.

Klir, G. J. and Folger, T. 1987. Fuzzy Sets, Uncertainty, and Information , Prentice Hall, Englewood Cliffs, NJ.

Klir, G. J. and Yuan, B. 1995. Fuzzy sets and fuzzy logic: Theory and Applications. Prentice Hall P T R, Upper Saddle River, New Jersey.

Kaufmann, A. and Gupta, M.M. (1985) Introduction to Fuzzy Arithmetic. Reinhold, New York.

Kotschy, K. A. (2003) Patch dynamics and clonal expansion of reeds in the Letea, M.Sc. Thesis University of the Witwatersrand.

Michaelides, K. and Wainwright, J. 2004. Modelling fluvial processes and interactions. In Environmental modelling: Finding simplicity in complexity. Edited by Wainwright, J. and Mulligan M., John Wiley & Sons, Ltd.

Murray, A. B. 2003. Contrasting the Goals, Strategies, and Predictions Associated With Simplified Numerical Models and Detailed Simulations. Geophysical Monograph 135, American Geophysical Union, 10.1029/135GM11.

Murray, A. B. and Paola, C. 1994. A cellular model of braided rivers. Letters to nature. Vol 371 pg 54 – 57.

Nield, J. M., Walker, D. J. and Lambert, M. F. 2002. Self-organisation and entropy in the 2-D morphological modelling of an open channel. University of Adelaide, Adelaide.

Werner, B.T. 1999. Complexity in natural landform patterns, Science, Vol 284, 102-104.

Wiens, J. A. 1998. Spatial scaling in ecology. Functional ecology. Vol 3, 385 – 397.

Zhang, X., Drake, N. A. and Wainright, J. (2004) Scaling issues in environmental modelling. In Environmental modelling: Finding simplicity in complexity. Edited by Wainwright J. and Mulligan M, John Wiley & Sons, Ltd.